Performance of Viscoelastically Damped Multilayer Structures Subjected to Shock Excitation

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Theme

THIS paper compares the shock response and damping effectiveness of viscoelasticity damped two-, three-, and five-layer simply supported sandwich beams (Fig. 1) subjected to half-sine-type shock excitation (Fig. 2). The comparison is made on constant size-and-weight criterion to study their relative performance. The computations are based on the generalized analysis developed in Ref. 1 in which dynamic properties of the viscoelastic material are represented (as in Ref. 2) by four-element viscoelastic model (Fig. 3); bending rigidity of the faces and rotary and logitudinal inertias of all the layers are taken into account. In the previous analyses for the shock response of multilayer structures, either a more restrictive viscoelastic model has been assumed,[3] the above mentioned inertias have been neglected, [3,4] or the faces have been assumed as membranes[5] only. Thus the results presented here are based on a more refined analysis.

Contents

The underlying assumption for the analysis in Ref. 1 are a) transverse sections of the face layers 1, 3, 5 in the five-layer beam, layers 1 and 3 in the three-layer beam, and both layers in the two-layer beam (Fig. 1) remain plane and normal to the longitudinal fibers of the respective layers before and after bending, shear deformation being neglected in all layers; b) at a given section, transverse displacement remains constant throughout the beam thickness; c) all displacements are assumed to be small; d) there is no interfacial slip; e) viscoelastic material is linear; f) strain energy due to longitudinal extension and bending is neglected for core layers 2 and 4 in the five-layer beam and layer 2 in the three-layer beam; g) longitudinal displacements u_i vary as shown in Fig. 1.

The solutions for transverse displacement response of a simply supported five-layer beam subjected to half-sine shock loading as obtained during and after the pulse era, respectively, are as follows.[1] The loading is applied to both ends of the beam simultaneously (as in a standard drop test)

$$\bar{W}(\bar{x},t) = \sum_{n=1,3}^{\infty} \left[\sum_{i=1}^{10} J_i e^{\Lambda_i t} + A \cos \omega t + B \sin \omega t \right] \sin n\pi\bar{x}$$

and

$$\bar{w}(\bar{x},t) = \sum_{n=1,3}^{\infty} \left[\sum_{i=1}^{10} J_i e^{\Lambda_i t} (1+e^{-\Lambda_i \tau}) \right] \sin n\pi\bar{x}$$

where $\bar{w}=w/L$, $\bar{x}=x/L$, t is the time, τ is the shock duration, Λ_i is ith root of 10th-order polynominal in the analysis, J_i, A, and B are constants depending upon the data, w is the transverse displacement, L is the beam length, ω is the angular frequency in rps, and x is the space coordinate in the longitudinal direction. Solutions in a similar form as above can be obtained[1] for the shock response of a simply supported three-layer beam with viscoelastic core and for a simply supported two-layer beam with one of the layers as viscoelastic (the top layer in the present case). These solutions are employed to calculate shock response $\bar{x}$ in the case of different beams.

Width, length, total thickness, and materials have been kept identical for two- three-, and five-layer simply supported beams while comparing their performance. Let t_e and t_v be the respective thickness of the elastic and viscoelastic layers of the two-layer beam. The corresponding three- and five-layer beams are made symmetrical in configuration having total thickness of elastic and viscoelastic layers as t_e and t_v, respectively.

The data used for Figs. 4 and 5 are width = 5 cm, $L=50$ cm, $t_e=0.5$ cm, thickness ratio $t_v/t_e=5.0$, $\int_e=0.28\times10^{-5}$ kg – sec^2/cm^4, $\int_e/\int_e=0.5$, $v=20$ cm/sec, $E_e=7\times10^5$ kg/cm^2, $\tau=\pi/500$ sec, where $\int_e$ and $\int_v$ are the mass densities/unit volume for elastic and viscoelastic materials, respectively, v = velocity of impact, and E_e is Young's modulus of the elastic material. Typical values of the model constants for the viscoelastic material in shear (three-layer and five-layer beams) are $\eta_2=$ 0.0282 kg-sec/cm^2, $\eta_3=0.0225$ kg-sec/cm^2, $\zeta_3=84.5$ kg/cm^2; ζ_1, which represents the static modulus, is kept variable. Assuming the viscoelastic material to be incompressible, the values of the model constants for the material in direct strain (a two-layer beam) can be shown[1] to be three times the above values. Thus $E^*=3G^*$ where E^* is the ratio of ζ_1 and E_e when ζ_1 refers to the material in direct strain and G^* is this ratio when ζ_1 refers to the material in shear strain. The term $\overline{w_{cp}}$ in Fig. 4 represents the peak value of $\bar{w}$ at the center of the beam span. Log-decrement δ in Fig. 5 has been taken as the natural logarithm of the ratio of two successive transverse displacement amplitudes of like sign occurring immediately after the disappearance of the shock pulse. Figures 6 and 7 have been plotted taking E^* ($=3G^*$) equal to 1.5×10^{-2} and 1.5×10^{-4}, respectively, keeping thickness ratio t_v/t_e as variable and using all other parameters as in Figs. 4 and 5. In all these figures, the symbols 2 – L, 3 – L, and 5 – L, stand for two-, three-, and five-layer simply supported beams, respectively.

It is seen that in the case of the two-layer beam with an unconstrained viscoelastic layer, peak displacement response is the highest (Figs. 4 and 6) and the damping effectiveness is the least (Figs. 5 and 7). Figures 6 and 7 show that the thickness ratio t_v/t_e has to be relatively large in the case of the two-layer beam to achieve any significant reduction in peak displacement response or any significant gain in damping ef-

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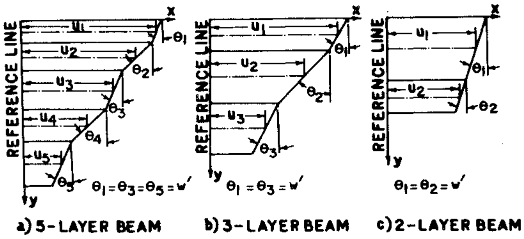

Fig. 1 Longitudinal displacements (magnitudes only).

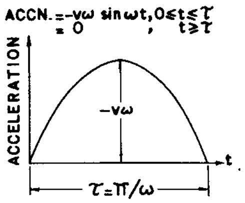

Fig. 2 Half-sine wave pulse acceleration.

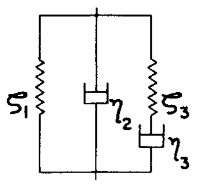

Fig. 3 Four-element viscoelastic model.

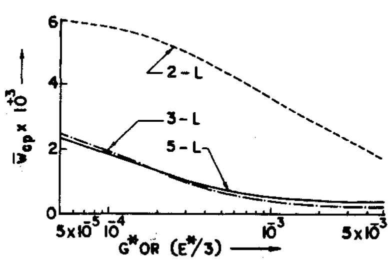

Fig. 4 Variation of $\overline{\omega}_{cp}$ with G^* and E^*.

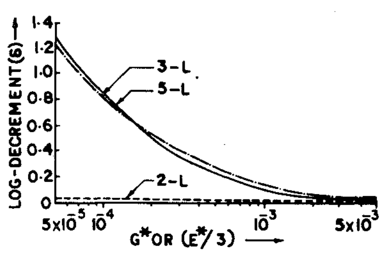

Fig. 5 Variation of δ with G^* and E^*.

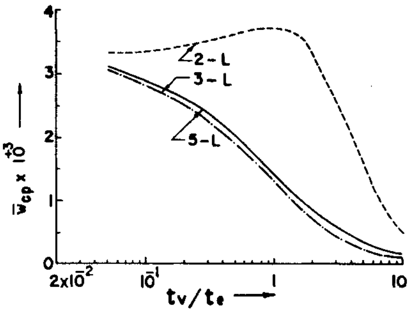

Fig. 6 Variation of $\overline{\omega}_{cp}$ with thickness ratio.

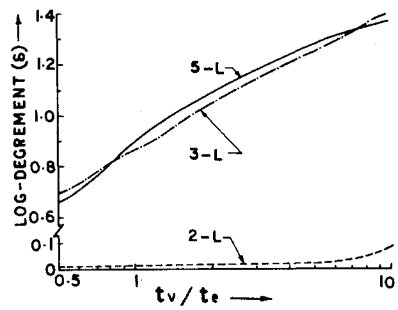

Fig. 7 Variation of δ with thickness ratio.

fectiveness. Furthermore, comparing the performance of the five-layer arrangement with that of a corresponding three-layer one, it is seen (Figs. 4-7) that any one of these may perform better depending upon the values of parameters G^* and t_v/t_e. The present study, however, indicates that a five-layer arrangement shows better effectiveness in peak displacement response (Fig. 4) and damping (Fig. 5) when G^* is smaller i.e., viscoelastic material is relatively softer.

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